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ON THE HISTORY OF GUNTER'S SCALE AND THE SLIDE RULE DURING THE SEVENTEENTH CENTURY

BY

FLORIAN CAJORI

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I. INTRODUCTION

In my history of the slide rule[1], and my article on its invention[2] it is shewn that William Oughtred and not Edmund Wingate is the inventor, that Oughtred's circular rule was described in print in 1632, his rectilinear rule in 1633. Richard Delamain is referred to as having tried to appropriate the invention to himself[3] and as having written a scurrilous pamphlet against Oughtred. All our information

[1] F. Cajori, *History of the Logarithmic Slide Rule and Allied Instruments*, New York, 1909, pp. 7–14, also Addenda i–vi.

[2] F. Cajori, "On the Invention of the Slide Rule," in *Colorado College Publication*, Engineering Series Vol. 1, 1910. An abstract of this is given in *Nature* (London), Vol. 82, 1909, p. 267.

[3] F. Cajori, *History* etc., p. 14.

about Delamain was taken from De Morgan,[4] who, however, gives no evidence of having read any of Delamain's writings on the slide rule. Through Dr. Arthur Hutchinson of Pembroke College, Cambridge, I learned that Delamain's writings on the slide rule were available. In this article will be given: First, some details of the changes introduced during the seventeenth century in the design of Gunter's scale by Edmund Wingate, Milbourn, Thomas Brown, John Brown and William Leybourn; second, an account of Delamain's book of 1630 on the slide rule which antedates Oughtred's first *publication* (though Oughtred's date of *invention* is earlier than the date of Delamain's alleged invention) and of Delamain's later designs of slide rules; third, an account of the controversy between Delamain and Oughtred; fourth, an account of a later book on the slide rule written by William Oughtred, and of other seventeenth century books on the slide rule.

II. INNOVATIONS IN GUNTER'S SCALE

We begin with Anthony Wood's account of Wingate's introduction of Gunter's scale into France.[5]

> In 1624 he transported into France the rule of proportion, having a little before been invented by Edm. Gunter of Gresham Coll. and communicated it to most of the chiefest mathematicians then residing in Paris: who apprehending the great benefit that might accrue thereby, importun'd him to express the use thereof in the French tongue. Which being performed accordingly, he was advised by monsieur Alleawne the King's chief engineer to dedicate his book to monsieur the King's only brother, since duke of Orleans. Nevertheless the said work coming forth as an abortive (the publishing thereof being somewhat hastened, by reason an advocate of Dijon in Burgundy began to print some uses thereof, which Wingate had in a friendly way communicated to him) especially in regard Gunter himself had learnedly explained its use in a far larger volume.[6]

Gunter's scale, which Wingate calls the "rule of proportion," contained, as described in the French edition of 1624, four lines: (1) A single line of numbers; (2) a line of tangents; (3) a line of sines; (4) a line, one foot in length, divided into 12 inches and tenths of inches, also a line, one foot in length, divided into tenths and hundredths.

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[4] Art. "Slide Rule" in the *Penny Cyclopaedia* and in the *English Cyclopaedia* [Arts and Sciences].

[5] Anthony Wood, *Athenae oxonienses* (Ed. P. Bliss), London, Vol. III, 1817, p. 423.

[6] The full title of the book which Wingate published on this subject in Paris is as follows:

L'Vsage|de la|Reigle de|Proportion|en l'Arithmetique &|Geometrie.|Par Edmond Vvingate,| Gentil-homme Anglois.|

E ἀνῆsφιλεμα τῆs, ἔση ηoλυμα τῆs.

In tenui, sed nõ tenuis vſuſve, laborne.|

A Paris,|Chez Melchior Mondiere,|demeurant en l'Isle du Palais,|à la|ruë de Harlay aux deux Viperes.|M. DC. XXIV.|Auec Priuilege du Roy.|

Back of the title page is the announcement:

> Notez que la Reigle de Proportion en toutes façons se vend à Paris chez Melchior Tauernier, Graueur & Imprimeur du Roy pour les Tailles douces, demeurant en l'Isle du Palais sur le Quay qui regarde la Megisserie à l'Espic d'or.

The English editions of this book which appeared in 1626 and 1628 are devoid of interest. The editions of 1645 and 1658 contain an important innovation.[7] In the preface the reasons why this instrument has not been used more are stated to be: (1) the difficulty of drawing the lines with exactness, (2) the trouble of working thereupon by reason (sometimes) of too large an extent of the compasses, (3) the fact that the instrument is not readily portable. The drawing of Wingate's arrangement of the scale in the editions of 1645 and 1658 is about 66 cm. (26.5 in.) long. It contains five parallel lines, about 66 cm. long, each having the divisions of one line marked on one side and of another line on the other side. Thus each line carries two graduations: (1) A single logarithmic line of numbers; (2) a logarithmic line of numbers thrice repeated; (3) the first scale repeated, but beginning with the graduations which are near the middle of the first scale, so that its graduation reads 4, 5, 6, 7, 8, 9, 1, 2, 3; (4) a logarithmic line of numbers twice repeated; (5) a logarithmic line of tangents; (6) a logarithmic line of sines; (7) the rule divided into 1000 equal parts; (8) the scale of latitudes; (9) a line of inches and tenths of inches; (10) a scale consisting of three kinds, viz., a gauge line, a line of chords, and a foot measure, divided into 1000 equal parts.

Important are the first and second scales, by which cube root extraction was possible "by inspection only, without the aid of pen or compass;" similarly the third and fourth scales, for square roots. This innovation is due to Wingate. The 1645 edition announces that the instrument was made in brass by Elias Allen, and in wood by John Thompson and Anthony Thompson in Hosier Lane.

William Leybourn, in his *The Line of Proportion or Numbers, Commonly called Gunter's Line, Made Easie*, London, 1673, says in his preface "To the Reader:"

> The Line of Proportion or Numbers, commonly called (by Artificers) Gunter's Line, hath been discoursed of by several persons, and variously applied to divers uses; for when Mr. Gunter had brought it from the Tables to a Line, and written some Uses thereof, Mr. Wingate added divers Lines of several lengths, thereby to extract the Square or Cube Roots, without doubling or trebling the distance of the Compasses: After him Mr. Milbourn, a Yorkshire Gentleman, disposed it in a Serpentine or Spiral Line, thereby enlarging the divisions of the Line.

On pages 127 and 128 Leybourn adds:

> Again, One T. Browne, a Maker of Mathematical Instruments, made it in a Serpentine or Spiral Line, composed of divers Concentrick Circles, thereby to enlarg the divisions, which was the contrivance of one Mr. Milburn a Yorkshire Gentleman, who writ thereof, and communicated his Uses to the aforesaid Brown, who (since his death) attributed it to himself: But whoever was the contriver of it, it is not without inconvenience; for it can in no wise be made portable; and besides (instead of compasses) an opening Joynt with thirds [threads] must be placed to move upon the Centre of the Instrument, without which no proportion can be wrought.

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[7] The title-page of the edition of 1658 is as follows:

> The Use of the Rule of Proportion in Arithmetick & Geometrie. First published at Paris in the French tongue, and dedicated to Monsieur, the then king's onely Brother (now Duke of Orleance). By Edm. Wingate, an English Gent. And now translated into English by the Author. Whereinto is now also inserted the Construction of the same Rule, & a farther use thereof . . . 2nd edition inlarged and amended. London, 1658.

This Mr. Milburn is probably the person named in the diary of the antiquarian, Elias Ashmole, on August 13 [1646?]; "I bought of Mr. Milbourn all his Books and Mathematical Instruments."[8] Charles Hutton[9] says that Milburne of Yorkshire designed the spiral form about 1650. This date is doubtless wrong, for Thomas Browne who, according to Leybourn, got the spiral form of line from Milbourn, is repeatedly mentioned by William Oughtred in his *Epistle*[10] printed some time in 1632 or 1633. Oughtred does not mention Milbourn, and says (page 4) that the spiral form "was first hit upon by one Thomas Browne a Joyner, . . . the serpentine revolution being but two true semicircles described on severall centers."[11]

Thomas Brown did not publish any description of his instrument, but his son, John Brown, published in 1661 a small book,[12] in which he says (preface) that he had done "as Mr. Oughtred with Gunter's Rule, to a sliding and circular form; and as my father Thomas Brown into a Serpentine form; or as Mr. Windgate in his *Rule of Proportion.*" He says also that "this brief touch of the Serpentine-line I made bold to assert, to see if I could draw out a performance of that promise, that hath been so long unperformed by the promisers thereof." Accordingly in Chapter XX he gives a description of the serpentine line, "contrived in five (or rather 15) turn." Whether this description, printed in 1661, exactly fits the instrument as it was developed in 1632, we have no means of knowing. John Brown says:

1. First next the center is two circles divided one into 60, the other into 100 parts, for the reducing of minutes to 100 parts, and the contrary.

2. You have in seven turnes two inpricks, and five in divisions, the first Radius of the sines (or Tangents being neer the matter, alike to the first three degrees,) ending at 5 degrees and 44 minutes.

3. Thirdly, you have in 5 turns the lines of numbers, sines, Tangents, in three margents in divisions, and the line of versed sines in pricks, under the line of Tangents, according to Mr. *Gunter's* cross-staff: the sines and Tangents beginning at 5 degrees, and 44 minutes where

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[8] *Memories of the Life of that Learned Antiquary, Elias Ashmole, Esq.*; *Drawn up by himself by way of Diary. With Appendix of original Letters.* Publish'd by Charles Burman, Esq., London, 1717, p. 23.

[9] *Mathematical Tables*, 1811, p. 36, and art. "Gunter's Line" in his *Phil. and Math. Dictionary*, London, 1815.

[10] *To the English Gentrie, and all others studious of the Mathematicks, which shall bee readers hereof. The just Apologie of Wil: Ovghtred, against the slaunderous insimulations of Richard Delamain, in a Pamphlet called Grammelogia, or the Mathematicall Ring, or Mirifica logarithmorum projectio circularis.* We shall refer to this document as *Epistle.* It was published without date in 32 unnumbered pages of fine print, and was bound in with Oughtred's *Circles of Proportion,* in the editions of 1633 and 1639. In the 1633 edition it is inserted at the end of the volume just after the *Addition vnto the Vse of the Instrument etc.*, and in that of 1639 immediately after the preface. It was omitted from the Oxford edition of 1660. The *Epistle* was also published separately. There is a separate copy in the British Museum, London. Aubrey, in his *Brief Lives,* edited by A. Clark, Vol. II, Oxford, 1898, p. 113, says quaintly, "He writt a stitch't pamphlet about 163(?4) against . . . Delamaine."

[11] Thomas Browne is mentioned by Stone in his *Mathematical Instruments,* London 1723, p. 16. See also Cajori, *History of the Slide Rule,* New York, 1909, p. 15.

[12] *The Description and Use of a Joynt-Rule*: . . . *also the use of Mr. White's Rule for measuring of Board and Timber, round and square*; *With the manner of Vsing the Serpentine-line of Numbers, Sines, Tangents, and Versed Sines.* By J. Brown, Philom., London, 1661.

the other ended, and proceeding to 90 in the sines, and 45 in the Tangents. And the line of numbers beginning at 10, and proceeding to 100, being one entire Radius, and graduated into as many divisions as the largeness of the instrument will admit, being 10 to 10 50 into 50 parts, and from 50 to 100 into 20 parts in one unit of increase, but the Tangents are divided into single minutes from the beginning to the end, both in the first, second and third Radiusses, and the sines into minutes; also from 30 minutes to 40 degrees, and from 40 to 60, into every two minutes, and from 60 to 80 in every 5th minute, and from 80 to 85 every 10th, and the rest as many as can be well discovered.

The versed sines are set after the manner of Mr. *Gunter's* Cross-staff, and divided into every 10th minutes beginning at 0, and proceeding to 156 going backwards under the line of Tangents.

4. Fourthly, beyond the Tangent of 45 in one single line, for one Turn is the secants to 51 degrees, being nothing else but the sines reitterated beyond 90.

5. Fifthly, you have the line of Tangents beyond 45, in 5 turnes to 85 degrees, whereby all trouble of backward working is avoided.

6. Sixthly, you have in one circle the 180 degrees of a Semicircle, and also a line of natural sines, for finding of differences in sines, for finding hour and Azimuth.

7. Seventhly, next the verge or outermost edge is a line of equal parts to get the Logarithm of any number, or the Logarithm sine and Tangent of any ark or angle to four figures besides the carracteristick.

8. Eightly and lastly, in the space place between the ending of the middle five turnes, and one half of the circle are three prickt lines fitted for reduction. The uppermost being for shillings, pence and farthings. The next for pounds, and ounces, and quarters of small *Averdupoies* weight. The last for pounds, shillings and pence, and to be used thus: If you would reduce 16s. 3d. 2q. to a decimal fraction, lay the hair or edge of one of the legs of the index on 16. $3\frac{1}{2}$ in the line of 1. s. d. and the hair shall cut on the equal parts 81 16; and the contrary, if you have a decimal fraction, and would reduce it to a proper fraction, the like may you do for shillings, and pence, and pounds, and ounces.

The uses of the lines follow.

As to the use of these lines, I shall in this place say but little, and that for two reasons. First, because this instrument is so contrived, that the use is sooner learned then any other, I speak as to the manner, and way of using it, because by means of first second and third radiusses, in sines and Tangents, the work is always right on, one way or other, according to the Canon whatsoever it be, in any book that treats of the Logarithms, as *Gunter, Wells, Oughtred, Norwood,* or others, as in *Oughtred* from page 64 to 107.

Secondly, and more especially, because the more accurate, and large handling thereof is more then promised, if not already performed by more abler pens, and a large manuscript thereof by my *Sires* meanes, provided many years ago, though to this day not extant in print; so for his sake I claiming my interest therein, make bold to present you with these few lines, in order to the use of them: And first note,

1. Which soever of the two legs is set to the first term in the question, that I call the first leg always, and the other being set to the second term, I call the second leg . . .

The exact nature of the contrivance with the "two legs" is not described, but it was probably a flat pair of compasses, attached to the metallic surface on which the serpentine line was drawn. In that case the instrument was a slide rule, rather than a form of Gunter's line. In his publication of 1661, as also in later

publications,[13] John Brown devoted more space to Gunter's scales, requiring the use of a separate pair of compasses, than to slide rules.

The same remark applies to William Leybourn who, after speaking of Seth Partridge's slide rule, returns to forms of Gunter's scale, saying:[14]

> There is yet another way of disposing of this Line of Proportion, by having one Line of the full length of the Ruler, and another Line of the same Radius broken in two parts between 3 and 4; so that in working your Compasses never go off of the Line: This is one of the best contrivances, but here Compasses must be used. These are all the Contrivances that I have hitherto seen of these Lines: That which I here speak of, and will shew how to use, is only two Lines of one and the same Radius, being set upon a plain Ruler of any length (the larger the better) having the beginning of one Line, at the end of the other, the divisions of each Line being set so close together, that if you find any number upon one of the Lines, you may easily see what number stands against it on the other Line. This is all the Variation. . . .
>
> *Example 1. If a Board be 1 Foot 64 parts broad, how much in length of that Board will make a Foot Square?* Look upon one of your Lines (it matters not which) for 1 Foot 64 parts, and right against it on the other Line you shall find 61; and so many parts of a Foot will make a Foot square of that Board.

This contrivance solves the equation $1.64x=1$, yielding centesimal parts of a foot.

James Atkinson[15] speaks of "Gunter's scale" as "usually of Boxwood . . . commonly 2 ft. long, $1\frac{1}{2}$ inch broad" and "of two kinds: *long Gunter* or *single Gunter*, and the *sliding Gunter*. It appears that during the seventeenth century (and long after) the Gunter's scale was a rival of the slide rule.

III. RICHARD DELAMAIN'S GRAMMELOGIA

We begin with a brief statement of the relations between Oughtred and Delamain. At one time Delamain, a teacher of mathematics in London, was assisted by Oughtred in his mathematical studies. In 1630 Delamain published the *Grammelogia*, a pamphlet describing a circular slide rule and its use. In 1631 he published another tract, on the *Horizontall Quadrant*.[16] In 1632 appeared Oughtred's *Circles of Proportion*[17] translated into English from Oughtred's Latin manuscript by another pupil, William Forster, in the preface of which Forster makes

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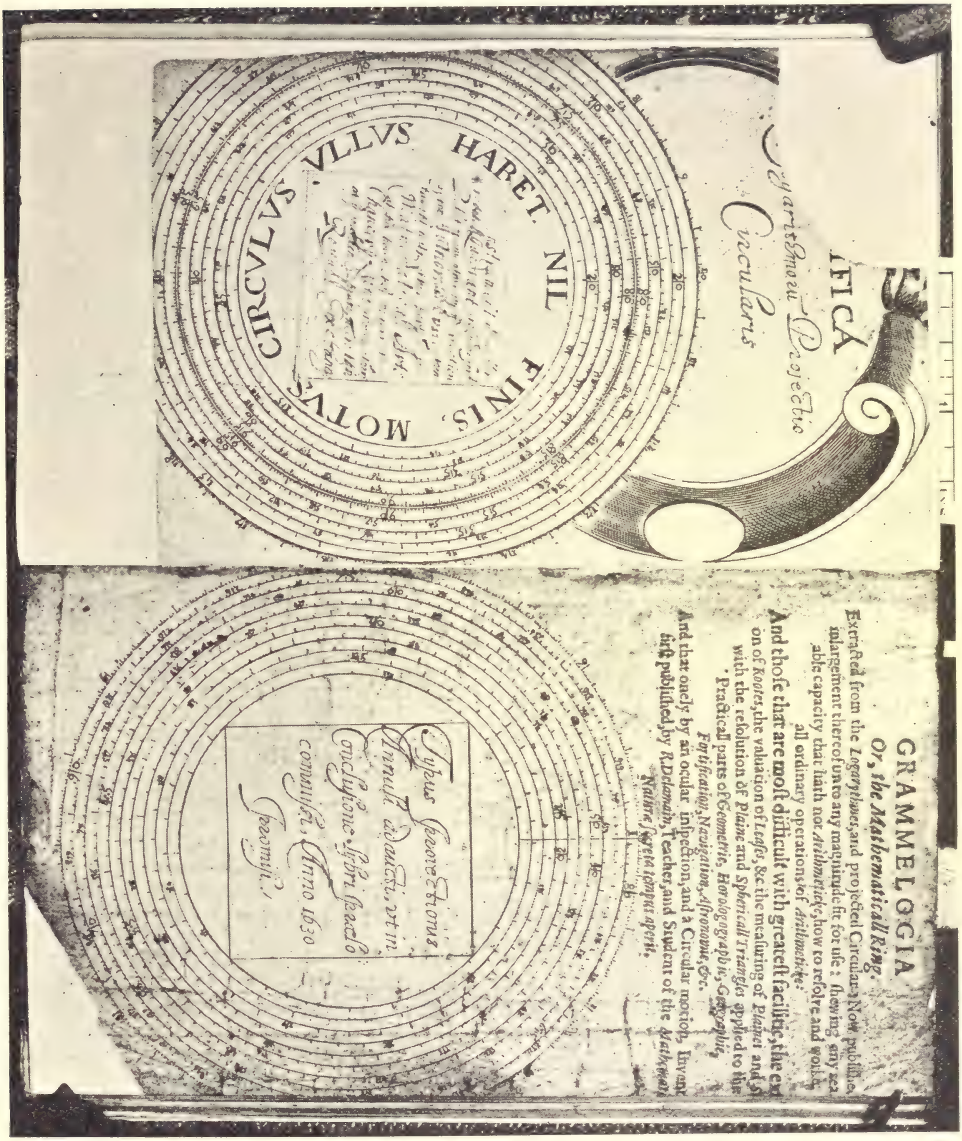
[13] *A Collection of Centers and Useful Proportions on the Line of Numbers*, by John Brown, 1662(?), 16 pages; *Description and Use of the Triangular Quadrant*, by John Brown, London, 1671; *Wingate's Rule of Proportion in Arithmetick and Geometry: or Gunter's Line. Newly rectified by Mr. Brown and Mr. Atkinson, Teachers of the Mathematicks*, London, 1683; *The Description and Use of the Carpenter's-Rule: Together with the Use of the Line of Numbers commonly call'd Gunter's-Line*, by John Brown, London, 1704.

[14] William Leybourn, *op. cit.*, pp. 129, 130, 132, 133.

[15] James Atkinson's edition of Andrew Wakely's *The Mariners Compass Rectified*, London, 1694 [Wakely's preface dated 1664, Atkinson's preface, 1693]. Atkinson adds *An Appendix containing Use of Instruments most useful in Navigation.* Our quotation is from this *Appendix*, p. 199.

[16] R. Delamain, *The Making, Description, and Use of a small portable Instrument . . . called a Horizontall Quadrant*, etc., London, 1631.

[17] Oughtred's description of his circular slide rule of 1632 and his rectilinear slide rule of 1633, as well as a drawing of the circular slide rule, are reproduced in Cajori's *History of the Slide Rule*, Addenda, pp. ii–vi.



The two title-pages of the edition of the GRAMMELOGIA in the British Museum in London which we have called "Grammelogia IV."

the charge (without naming Delamain) that "another . . . went about to preocupate" the new invention. This led to verbal disputes and to the publication by Delamain of several additions to the *Grammelogia*, describing further designs of circular slide rules and also stating his side of the bitter controversy, but without giving the name of his antagonist. Oughtred's *Epistle* was published as a reply. Each combatant accuses the other of stealing the invention of the circular slide rule and the horizontal quadrant.

There are at least five different editions, or impressions, of the *Grammelogia* which we designate, for convenience, as follows:

Grammelogia I, 1630. One copy in the Cambridge University Library.[17]
Grammelogia II, I have not seen a copy of this.
Grammelogia III, One copy in the Cambridge University Library.[18]
Grammelogia IV, One copy in the British Museum, another in the Bodleian Library, Oxford.[19]
Grammelogia V, One copy in the British Museum.

In *Grammelogia I* the first three leaves and the last leaf are without pagination. The first leaf contains the title-page; the second leaf, the dedication to the King and the preface "To the Reader;" the third leaf, the description of the Mathe-

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[17] The full title of the *Grammelogia I* is as follows:

Gram̄elogia|or,|The Mathematicall Ring.|Shewing (any reasonable Capacity that hath| not Arithmeticke) how to resolve and worke|all ordinary operations of Arithmeticke.| And those which are most difficult with greatest|facilitie: The extraction of Roots, the valuation of|Leases, &c. The measuring of Plaines|and Solids.|With the resolution of Plaine and Sphericall|Triangles.|And that onely by an Ocular Inspection,|and a Circular Motion.|Naturae secreta tempus aperit.|London printed by John Haviland, 1630.

[18] *Grammelogia III* is the same as *Grammelogia I*, except for the addition of an appendix, entitled:

De la Mains|Appendix|Vpon his|Mathematicall|Ring. Attribuit nullo (praescripto tempore) vitae|vsuram nobis ingeniique Deus.|London,|

. . . The next line or two of this title-page which probably contained the date of publication, were cut off by the binder in trimming the edges of this and several other pamphlets for binding into one volume.

[19] *Grammelogia IV* has two title pages. The first is *Mirifica Logarithmoru' Projectio Circularis*. There follows a diagram of a circular slide rule, with the inscription within the innermost ring: *Nil Finis, Motvs, Circvlvs vllvs Habet.* The second title page is as follows:

Grammelogia|Or, the Mathematicall Ring.|Extracted from the Logarythmes, and projected Circular: Now published in the|inlargement thereof unto any magnitude fit for use: shewing any reason-|able capacity that hath not Arithmeticke how to resolve and worke,| all ordinary operations of Arithmeticke:|And those that are most difficult with greatest facilitie, the extracti-|on of Rootes, the valuation of Leases, &c. the measuring of Plaines and Solids,|with the resolution of Plaine and Sphericall Triangles applied to the|Practicall parts of Geometrie, Horologographie, Geographie|Fortification, Navigation, Astronomie, &c.|And that onely by an ocular inspection, and a Circular motion, Invented and first published, by R. Delamain, Teacher, and Student of the Mathematicks.| Naturae secreta tempus aperit.|

There is no date. There follows the diagram of a second circular slide rule, with the inscription within the innermost ring: *Typus proiectionis Annuli adaucti vt in Conslusione Lybri praelo commissi, Anno 1630 promisi.* There are numerous drawings in the *Grammelogia*, all of which, excepting the drawings of slide rules on the engraved title-pages of *Grammelogia IV* and *V*, were printed upon separate pieces of paper and then inserted by hand into the vacant spaces on the printed pages reserved for them. Some drawings are missing, so that the Bodleian *Grammelogia IV* differs in this respect slightly from the copy in the British Museum and from the British Museum copy of *Grammelogia V*.

matical Ring. Then follow 22 numbered pages. Counting the unnumbered pages, there are altogether 30 pages in the pamphlet. Only the first three leaves of this pamphlet are omitted in *Grammelogia IV* and *V*.

In *Grammelogia III* the *Appendix* begins with a page numbered 52 and bears the heading "Conclusion;" it ends with page 68, which contains the same two poems on the mathematical ring that are given on the last page of *Grammelogia I* but differs slightly in the spelling of some of the words. The 51 pages which must originally have preceded page 52, we have not seen. The edition containing these we have designated *Grammelogia II*. The reason for the omission of these 51 pages can only be conjectured. In Oughtred's *Epistle* (p. 24), it is stated that Delamain had given a copy of the *Grammelogia* to Thomas Brown, and that two days later Delamain asked for the return of the copy, "because he had found some things to be altered therein" and "rent out all the middle part." Delamain labored "to recall all the bookes he had given forth, (which were many) before the sight of *Brownes Lines*." These spiral lines Oughtred claimed that Delamain had stolen from Brown. The title-page and page 52 are the only parts of the *Appendix*, as given in *Grammelogia III*, that are missing in the *Grammelogia IV* and *V*.

Grammelogia IV answers fully to the description of Delamain's pamphlet contained in Oughtred's *Epistle*. It was brought out in 1632 or 1633, for what appears to be the latest part of it contains a reference (page 99) to the *Grammelogia I* (1630) as "being now more then two yeares past." Moreover, it refers to Oughtred's *Circles of Proportion*, 1632, and Oughtred's reply in the *Epistle* was bound in the *Circles of Proportion* having the *Addition* of 1633. For convenience of reference we number the two title-pages of *Grammelogia IV*, "page (1)" and "page (2)," as is done by Oughtred in his *Epistle*. *Grammelogia IV* contains, then, 113 pages. The page numbers which we assign will be placed in parentheses, to distinguish them from the page numbers which are *printed* in *Grammelogia IV*. The pages (44)–(65) are the same as the pages 1–22, and the pages (68)–(83) are the same as the pages 53–68. Thus only thirty-eight pages have page numbers printed on them. The pages (67) and (83) are identical in wording, except for some printer's errors; they contain verses in praise of the *Ring*, and have near the bottom the word "Finis." Also, pages (22) and (23) are together identical in wording with page (113), which is set up in finer type, containing an advertisement of a part of *Grammelogia IV* explaining the mode of graduating the circular rules. There are altogether six parts of *Grammelogia IV* which begin or end by an address to the reader, thus: "To the Reader," "Courteous Reader," or "To the courteous and benevolent Reader . . .," namely the pages (8), (22), (68), (89), (90), (108). In his *Epistle* (page 2), Oughtred characterizes the make up of the book in the following terms:

> In reading it . . . I met with such a patchery and confusion of disjoynted stuffe, that I was striken with a new wonder, that any man should be so simple, as to shame himselfe to the world with such a hotch-potch.

Grammelogia V differs from *Grammelogia IV* in having only the second title-page. The first title-page may have been torn off from the copy I have seen. A second difference is that the page with the printed numeral 22 in *Grammelogia IV* has after the word "Finis" the following notice:

> This instrument is made in Silver, or Brasse for the Pocket, or at any other bignesse, over against Saint Clements Church without Temple Barre, by Elias Allen.

This notice occurs also on page 22 of *Grammelogia I* and *III*, but is omitted from page 22 of *Grammelogia V*.

In his address to King Charles I, in his *Grammelogia I*, Delamain emphasizes the ease of operating with his slide rule by stating that it is "fit for use . . . as well on Horse backe as on Foot." Speaking "To the Reader," he states that he has "for many yeares taught the Mathematicks in this Towne," and made efforts to improve Gunter's scale "by some Motion, so that the whole body of Logarithmes might move proportionally the one to the other, as occasion required. This conceit in February last [1629] I struke upon, and so composed my *Grammelogia* or *Mathematicall Ring*; by which only with an *ocular inspection*, there is had at one instant all proportionalls through the said body of Numbers." He dates his preface "first of January, 1630." The fifth and sixth pages contain his "Description of the Grammelogia," the term *Grammelogia* being applied to the instrument, as well as to the book. His description is as follows:

> The parts of the Instrument are two Circles, the one moveable, and the other fixed; The moveable is that unto which is fastened a small pin to move it by; the other Circle may be conceived to be fixed; The circumference of the moveable Circle is divided into unequall parts, charactered with figures thus, 1. 2. 3. 4. 5. 6. 7. 8. 9. these figures doe represent themselves, or such numbers unto which a Cipher or Ciphers are added, and are varied as the occasion falls out in *the speech of Numbers*, so 1. stands for 1. or 10. or 100., &c. the 2. stands for 2. or 20. or 200. or 2000., &c. the 3. stands for 30. or 300. or 3000., &c.

After elaborating this last point and explaining the decimal subdivisions on the scales of the movable circle, he says that "the numbers and divisions on the fixed Circle, are the very same that the moveable are, . ." There is no drawing of the slide rule in this publication. The twenty-two numbered pages give explanations of the various uses to which the instrument can be put: "How to performe the Golden Rule" (pp. 1–3), "Further uses of the Golden Rule" (pp. 4–6), "Notions or Principles touching the disposing or ordering of the Numbers in the Golden Rule in their true places upon the Grammelogia" (pp. 7–11), "How to divide one number by another" (pp. 12, 13), "to multiply one Number by another" (pp. 14, 15), "To find Numbers in continuall proportion" (pp. 16, 17), "How to extract the Square Root," "How to extract the Cubicke Root" (pp. 18–21), "How to performe the Golden Rule" (the rule of proportion) is explained thus:

> Seeke the first number in the moveable, and bring it to the second number in the fixed, so right against the third number in the moveable, is the answer in the fixed.
>
> If the Interest of 100. li. be 8. li. in the yeare, what is the Interest of 65. li. for the same time.
>
> Bring 100. in the moveable to 8. in the fixed, so right against 65. in the moveable is 5.2. in the fixed, and so much is the Interest of 65. li. for the yeare at 8. li. for 100. li. *per annum*.

The *Instrument* not removed, you may at one instant right against any summe of money in the moveable, see the Interest thereof in the fixed: the reason of this is from the *Definition of Logarithmes.*

These are the earliest known printed instructions on the use of a slide rule. It will be noticed that the description of the instrument at the opening makes no references to logarithmic lines for the trigonometric functions; only the line of numbers is given. Yet the title-page promised the "resolution of Plaine and Sphericall Triangles." Page 22 throws light upon this matter:

If there be composed three Circles of equal thicknesse, A.B.C. so that the inner edge of D [should be B] and the outward egde of A bee answerably graduated with *Logarithmall signes* [sines], and the outward edge of B and the inner edge of A with *Logarithmes*; and then on the backside be graduated the *Logarithmall Tangents,* and againe the *Logarithmall signes* oppositly to the former graduations, it shall be fitted for the resolution of *Plaine* and *Sphericall Triangles.*

After twelve lines of further remarks on this point he adds:

Hence from the forme, I have called it a *Ring,* and *Grammelogia* by annoligie of a *Lineary speech*; which *Ring,* if it were projected in the *convex* unto two yards *Diameter,* or thereabouts, and the line *Decupled,* it would worke *Trigonometrie* unto seconds, and give *proportionall numbers* unto six places only by an *ocular inspection,* which would compendiate *Astronomicall calculations,* and be sufficient for the *Prosthaphaeresis* of the Motions: But of this as God shall give life and ability to health and time.

The unnumbered page following page 22 contains the patent and copyright on the instrument and book:

Whereas Richard Delamain, Teacher of Mathematicks, hath presented vnto Vs an Instrument called Grammelogia, or The Mathematicall Ring, together with a Booke so intituled, expressing the use thereof, being his owne Invention; we of our Gracious and Princely favour have granted unto the said Richard Delamain and his Assignes, Privilege, Licence, and Authority, for the sole Making, Printing and Selling of the said Instrument and Booke: straightly forbidding any other to Make, Imprint, or Sell, or cause to be Made, or Imprinted, or Sold, the said Instrument or Booke within any our Dominions, during the space of ten yeares next ensuing the date hereof, upon paine of Our high displeasure. Given under our hand and Signet at our Palace of Westminster, the fourth day of January, in the sixth yeare of our Raigne.

In the *Appendix* of *Grammelogia III,* on page 52 is given a description of an instrument promised near the end of *Grammelogia I*:

That which I have formerly delivered hath been onely upon one of the *Circles* of my *Ring,* simply concerning *Arithmeticall Propo tions,* I will by way of *Conclusion* touch upon some uses of the *Circles,* of *Logarithmall Sines,* and *Tangents,* which are placed on the edge of both the moveable and fixed *Circles* of the *Ring* in respect of *Geometricall Proportions,* but first of the description of these *Circles.*

First, upon the side that the *Circle of Numbers* is one, are graduated on the edge of the moveable, and also on the edge of the fixed the *Logarithmall Sines,* for if you bring 1. in the moveable amongst the *Numbers* to 1. in the fixed, you may on the other edge of the moveable and fixed see the *sines* noted thus 90. 90. 80. 80. 70. 70. 60. 60. &c. unto 6.6. and each degree subdivided, and then over the former divisions and figures 90. 90. 80. 80. 70. 70. &c. you have the other degrees, viz. 5. 4. 3. 2. 1. each of those divided by small points.

Secondly, (if the *Ring* is great) neere the outward edge of this side of the fixed against the *Numbers,* are the usuall divisions of a *Circle,* and the points of the *Compasse*: serving for observation in *Astronomy,* or *Geometry,* and the *sights* belonging to those divisions, may be placed on the moveable *Circle.*

Thirdly, opposite to those *Sines* on the other side are the *Logarithmall Tangents,* noted a like both in the moveable and fixed thus 6.6.7.7.8.8.9.9.10.10.15.15.20.20. &c. unto 45.45. which numbers or divisions serve also for their *Complements* to 90. so 40 gr. stands for 50. gr. 30. gr. for 60 gr. 20. gr. for 70. gr. &c. each degree here both in the moveable and fixed is also divided into parts. As for the degrees which are under 6. viz. 5.4.3.2.1. they are noted with small figures over this divided *Circle* from 45.40.35.30.25. &c. and each of those degrees divided into parts by small points both in the moveable and fixed.

Fourthly, on the other edge of the moveable on the same side is another graduation of *Tangents,* like that formerly described. And opposite unto it, in the fixed is a Graduation *of Logarithmall sines* in every thing answerable to the first descrition of Sines on the other side.

Fifthly, on the edge of the *Ring* is graduated a parte of the *Æquator,* numbered thus 10 20. 30. unto 100. and there unto is adjoyned the degrees of the *Meridian* inlarged, and numbred thus 10 20.30 unto 70. each degree both of the *Æquator,* and *Meridian* are subdivided into parts; these two graduated *Circles* serve to resolve such *Questions* which concerne *Latitude, Longitude, Rumb,* and *Distance,* in *Nauticall* operations.

Sixthly, to the concave of the *Ring* may be added a *Circle* to be elevated or depressed for any *Latitude,* representing the *Æquator,* and so divided into houres and parts with an *Axis,* to shew both the *houre,* and *Azimuth.* and within this *Circle* may be hanged a *Box,* and *Needle* with a Socket for a *staffe* to slide into it, and this accommodated with *scrue pines* to fasten it to the *Ring* and *staffe,* or to take it off at pleasure.

The pages bearing the printed numbers 53–68 in the *Grammelogia III, IV* and *V* make no reference to the dispute with Oughtred and may, therefore, be assumed to have been published before the appearance of Oughtred's *Circles of Proportion.* On page 53, "To the Reader," he says:

. . . you may make use of the Projection of the *Circles* of the *Ring* upon a *Plaine,* having the feet of a paire of *compasses* (but so that they be flat) to move on the *Center* of that *Plaine,* and those feet to open and shut as a paire of *Compasses* . . . now if the feet bee opened to any two termes or numbers in that *Projection,* then may you move the first foot to the third number, and the other foot shall give the *Answer*; . . . it hath pleased some to make use of this way. But in this there is a double labour in respect to that of the *Ring,* the one in fitting those feet unto the numbers assigned, and the other by moving them about, in which a man can hardly accommodate the *Instrument* with one hand, and expresse the *Proportionals* in writing with the other. By the *Ring* you need not but bring one number to another, and right against any other number is the *Answer* without any such motion. . . . upon that [the Ring] I write, shewing some uses of those *Circles* amongst themselves, and conjoyned with others . . . in *Astronomy, Horolographie,* in plaine *Triangles* applyed to *Dimensions, Navigation, Fortification,* etc. . . . But before I come to *Construction,* I have thought it convenient by way introduction, to examine the truth of the graduation of those *Circles* . . .

These are the words of a practical man, interested in the mechanical development of his instrument. He considers not only questions of convenience but also of accuracy. The instrument has, or may have now, also lines of sines and tangents. To test the accuracy of the circles of Numbers, "bring any number in the moveable to halfe of that number in the fixed: so any number or part in the fixed shall give his double in the moveable, and so may you trie of the thirds, fourths &c. of num-

bers, *vel contra*," (p. 54). On page 55 are given two small drawings, labelled, "A Type of the Ringe and Scheme of this Logarithmicall projection, the use followeth. These Instruments are made in Silver or Brasse by John Allen neare the Sauoy in the Strand."

IV. CONTROVERSY BETWEEN OUGHTRED AND DELAMAIN ON THE INVENTION OF THE CIRCULAR SLIDE RULE

Delamain's publication of 1630 on the 'Mathematicall Ring' does not appear at that time to have caused a rupture between him and Oughtred. When in 1631 Delamain brought out his *Horizontall Quadrant*, the invention of which Delamain was afterwards charged to have stolen from Oughtred, Delamain was still in close touch with Oughtred and was sending Oughtred in the Arundell House, London, the sheets as they were printed. Oughtred's reference to this in his *Epistle* (p. 20) written after the friendship was broken, is as follows:

> While he was printing his tractate of the Horizontall quadrant, although he could not but know that it was injurious to me in respect of my free gift to Master *Allen*, and of *William Forster*, whose translation of my rules was then about to come forth: yet such was my good nature, and his shamelessnesse, that every day, as any sheet was printed, hee sent, or brought the same to mee at my chamber in Arundell house to peruse which I lovingly and ingenuously did, and gave him my judgment of it.

Even after Forster's publication of Oughtred's *Circles of Proportion*, 1632, Oughtred had a book, *A canon of Sines Tangents and Secants*, which he had borrowed from Delamain and was then returning (*Epistle*, page (5)). The attacks which Forster, in the preface to the *Circles of Proportion*, made upon Delamain (though not naming Delamain) started the quarrel. Except for Forster and other pupils of Oughtred who urged him on to castigate Delamain, the controversy might never have arisen. Forster expressed himself in part as follows:

> . . . being in the time of the long vacation 1630, in the Country, at the house of the Reverend, and my most worthy friend, and Teacher, Mr. William Oughtred (to whose instruction I owe both my initiation, and whole progresse in these Sciences.) I vpon occasion of speech told him of a Ruler of Numbers, Sines, & Tangents, which one had be-spoken to be made (such as it vsually called Mr. Gunter's Ruler) 6 feet long, to be vsed with a payre of beame-compasses. "He answered that was a poore invention, and the performance very troublesome: But, said he, seeing you are taken with such mechanicall wayes of Instruments, I will shew you what deuises I have had by mee these many yeares." And first, hee brought to mee two Rulers of that sort, to be vsed by applying one to the other, without any compasses: and after that hee shewed mee those lines cast into a circle or Ring, with another moueable circle vpon it. I seeing the great expeditenesse of both those wayes; but especially, of the latter, wherein it farre excelleth any other Instrument which hath bin knowne; told him, I wondered that hee could so many yeares conceale such vseful inuentions, not onely from the world, but from my selfe, to whom in other parts and mysteries of Art, he had bin so liberall. He answered, "That the true way of Art is not by Instruments, but by Demonstration: and that it is a preposterous course of vulgar Teachers, to begin with Instruments, and not with the Sciences, and so in-stead of Artists, to make their Schollers only doers of tricks, and as it were Iuglers: to the despite of Art, losse of precious time, and betraying of willing and industrious wits,

vnto ignorance and idlenesse. That the vse of Instruments is indeed excellent, if a man be an Artist: but contemptible, being set and opposed to Art. And lastly, that he meant to commend to me, the skill of Instruments, but first he would haue me well instructed in the Sciences. He also shewed me many notes, and Rules for the vse of those circles, and of his Horizontall Instrument, (which he had proiected about 30 yeares before) the most part written in Latine. All which I obtained of him leaue to translate into English, and make publique, for the vse, and benefit of such as were studious, and louers of these excellent Sciences.

Which thing while I with mature, and diligent care (as my occasions would give me leaue) went about to doe: another to whom the Author in a louing confidence discouered this intent, using more hast then good speed, went about to preocupate; of which vntimely birth, and preuenting (if not circumuenting) forwardnesse, I say no more: but aduise the studious Reader, onely so farre to trust, as he shal be sure doth agree to truth & Art.

While in this dedication reference is made to a slide rule or "ring" with a "moveable circle," the instrument actually described in the *Circles of Proportion* consists of fixed circles "with an *index* to be opened after the manner of a paire of Compasses." Delamain, as we have seen, had decided preference for the moveable circle. To Oughtred, on the other hand, one design was about as good as the other; he was more of a theorist and repeatedly expressed his contempt for mathematical instruments. In his *Epistle* (page (25)), he says he had not "the one halfe of my intentions upon it" (the rule in his book), nor one with a "moveable circle and a thread, but with an opening Index at the centre (if so be that bee cause enough to make it to bee not the same, but another Instrument) for my part I disclaime it: it may go seeke another Master: which for ought I know, will prove to be *Elias Allen* himselfe: for at his request only I altered a little my rules from the use of the moveable circle and the thread, to the two armes of an Index."

All parts of Delamain's *Grammelogia IV*, except pages 1–22 and 53–68 considered above, were published after the *Circles of Proportion*, for they contain references to the ill treatment that Delamain felt or made believe that he felt, that he had received in the book published by Oughtred and Forster. Oughtred's reference to teachers whose scholars are "doers of tricks," "Iuglers," and Forster's allusion to "another to whom the Author in a loving confidence" explained the instrument and who "went about to preocupate" it, are repeatedly mentioned. Delamain says, (page (89)) that at first he did not intend to express himself in print, "but sought peace and my right by a private and friendly way." Oughtred's account of Delamain's course is that of an "ill-natured man" with a "virulent tongue," "sardonical laughter" and "malapert sawsiness." Contrasting Forster and Delamain, he says that, of the former he "had the very first moulding" and made him feel that "the way of Art" is "by demonstration." But Delamain was "already corrupted with doing upon Instruments, and quite lost from ever being made an Artist." (*Epistle* page (27)). Repeatedly does Oughtred assert Delamain's ignorance of mathematics. The two men were evidently of wholly different intellectual predilections. That Delamain loved instruments is quite evident, and we proceed to describe his efforts to improve the circular slide rule.

The *Grammelogia IV* is dedicated to King Charles I. Delamain says:

> . . . Everything hath his beginning, and curious *Arts* seldome come to the height at the first; It was my promise then to enlarge the *invention* by a way of *decuplating the Circles*, which I now present unto your *sacred Majestie* as the *quintessence* and *excellencie* there of . . .

His enlarged circular rules are illustrated in the Bodleian Library copy of *Grammelogia IV* by four diagrams, two of them being the two drawings on the two title-pages at the beginning of the *Grammelogia IV*, 4 inches in external diameter, and exhibiting eleven concentric circular lines carrying graduations of different sorts. In the second of these designs all circles are fixed. The other two drawings are each 10¾ inches in external diameter and exhibit 18 concentric circular lines; the folded sheet of the first of these drawings is inserted between pages (23) and (24), the second folded sheet between pages (83) and (84). All circles of this second instrument are fixed. Counting in the two small drawings in *Grammelogia III*, there are in all six drawings of slide rules in the Bodleian *Grammelogia IV*. On pages (24) to (43) Delamain explains the graduation of slide rules. He takes first a rule which has one circle of equal parts, divided into 1000 equal divisions. From a table of logarithms he gets $\log 2 = 0.301$; from the number 301 in the circle of equal parts he draws a line to the center of the circle and marks the intersection with the circles of numbers by the figure 2. Thus he proceeds with log 3, log 4, and so on; also with $\log \sin x$ and $\log \tan x$. For $\log \sin x$ he uses two circles, the first (see page (27)) for angles from $34' \, 24''$ to $5° \, 44' \, 22''$, the second circle from $5° \, 44' \, 22''$ to $90°$. The drawings do not show the seconds. He suggests many different designs of rules. On page (29) he says:

> For the single projection of the *Circles of my Ring*, and the dividing and graduating of them: which may bee so inserted upon the edges of *Circles* of mettle turned in the forme of a *Ring*, so that one *Circle* may moove betweene two fixed, by helpe of two stayes, then may there be graduated on the *face of the Ring*, upon the outer edge of the mooveable and inner edge of the fixed, the *Circle of Numbers*, then upon the inner edge of that mooveable *Circle*, and the outward edge of that inner fixed Circle may be inserted the *Circle of Sines*, and so according to the description of those that are usually made.

In addition to these lines he proceeds to mention the circle giving the ordinary division into degrees and minutes, and two circles of tangents on the other side of the rule.

Next Delamain explains an arrangement of all the graduation on one side of the rule by means of "a small channell in the innermost *fixed Circle*, in which may be placed a small single Index, which may have sufficient length to reach from the innermost edge of the *Mooveable Circle*, unto the outmost edge of the *fixed Circle*, which may be mooved to and fro at pleasure, in the channell, which Index may serve to shew the opposition of Numbers" (p. (31)). From this it is clear that the invention of the "runner" goes back to the very first writers on the slide rule.

After describing a modification of the above arrangement, he adds, "many other formes might be deliverd, about this single *projection*" (p. (32)).

Proceeding to the "enlarging" of the circles in the Ring, to, say, the "*Quadruple* to that which is single, that is, foure times greater," the "equall parts" are distributed over four circles instead of only one circle, but the general method of graduation is the same as before (p. (33)); there being now four circles carrying the logarithms of numbers, and so on. Next he points out "severall wayes how the Circles of the Mathematicall Ring (being inlarged) may be accommodated for practicall use:" (1) The Circles are all fixed in a plain and movable flat compasses (or better, a movable semicircle) are used for fixing any two positions; (2) There is a "double projection" of each logarithmic line "inlarged on a Plaine," one fixed, the other movable, as shown in his first figure on the title-page, a single index only being used; (3) use of "my great *Cylinder* which I have long proposed (in which all the Circles are of equall greatnesse,) and it may be made of any magnitude or capacity, but for a study (hee that will be at the charge) it may be of a yard diameter and of such an indifferent length that it may containe 100 or more Circles fixed parallel one to the other on the *Cylinder*, having a space betweene each of them, so that there may bee as many mooveable Circles, as there are fixed ones, and these of the mooveable linked, or fastened together, so that they may all moove together by the fixed ones in these spaces, whose edges both of the fixed, and mooveable being graduated by helpe of a single Index will shew the proportionalls by opposition in this double *Projection*, or by a double *Index* in a single *Projection*" (p. (36)).

Next follows the detailed description of his Ring "on a Plaine, according to the diagramme that was given the King (for a view of that projection) and afterwards the Ring it selve." The diagram is the large one which we mentioned as inserted between pages (23) and (24). The instrument has two circles, one moveable, upon each of which are described 13 distinct circular graduations. The lines on the fixed circle are: "The Circle of degrees and calendar," E. "Circle of equall parts, and part of the Equator, and Meridian," TT. "The Circle of Tangents," S. "The Circle of Sines," D. "The Circle of Decimals," N. "The Circle of Numbers." The lines on the movable circle are: N. "The Circle of Numbers," E. "The Circle of equated figures, and bodies," S. "The Circle of Sines," TT. "The Circle of Tangents," Y. "The Circle of time, yeares, and monethes."

On pages (84)–(88) Delamain explains an enlargement of his Ring for computations involving the sines of angles near to 90°. On page (86) he says:

> I have continued the *Sines* of the *Projection* unto two severall *revolutions*, the one beginning at 77.gr. 45.m. 6.s. and ends at 90.gr. (being the last *revolution* of the *decuplation* of the former, or the hundred part of that *Projection*) the other beginning at 86.gr. 6.m. 48.s. and ends at 90.gr. (being the last of a ternary of *decuplated revolutions*, or the thousand part of that *Projection*) and may bee thus used.

He explains the manner of using these extra graduations. Thus he claims to have attained degrees of accuracy which enabled him to do what "some one" had declared "could not bee done." It is hardly necessary to point out that Delamain's *Grammelogia IV* suggests designs of slide rules which inventors two hundred or more years later were endeavouring to produce. Which of Delamain's designs

of rules were actually made and used, he does not state explicitly. He refers to a rule 18 inches in diameter as if it had been actually constructed (pages (86), (88)). Oughtred showed no appreciation of such study in designing and ridiculed Delamain's efforts, in his *Epistle.*

Additional elucidations of his designs of rules, along with explanations of the relations of his work to that of Gunter and Napier, and sallies directed against Oughtred and Forster, are contained on pages (8)–(21) of his *Grammelogia IV.*

V. INDEPENDENCE AND PRIORITY OF INVENTION

The question of independence and priority of invention is discussed by Delamain more specifically on pages (89)–(113); Oughtred devotes his entire *Epistle* to it. It is difficult to determine definitely which publication is the later, Delamain's *Grammelogia IV* or Oughtred's *Epistle.* Each seems to quote from the other. Probably the explanation is that the two publications contain arguments which were previously passed from one antagonist to the other by word of mouth or by private letter. Oughtred refers in his *Epistle* (p. (12)) to a letter from Delamain. We believe that the *Epistle* came after Delamain's *Grammelogia IV.* Delamain claims for himself the invention of the circular slide rule. He says in his *Grammelogia IV.* (p. (99)), "when I had a sight of it, which was in *February,* 1629 (as I specified in my *Epistle*) I could not conceale it longer, envying my selfe, that others did not tast of that which I found to carry with it so delightfull and pleasant a goate [taste] . . ." Delamain asserts (without proof) that Oughtred "never saw it as he now challengeth it to be his invention, untill it was so fitted to his hand, and that he made all his practise on it after the publishing of my *Booke* upon my *Ring,* and not before; so it was easie for him or some other to write some uses of it in Latin after Christmas, 1630 and not the *Sommer* before, as is falsely alledged by some one . . ." (p. (91)). Delamain's accusation of theft on the part of Oughtred cannot be seriously considered. Oughtred's reputation as a mathematician and his standing in his community go against such a supposition. Moreover, William Forster is a witness for Oughtred. The fact that Oughtred had the mastery of the rectilinear slide rule as well, while Delamain in 1630 speaks only of the circular rule, weighs in Oughtred's favour.

Oughtred says he invented the slide rule "above twelve yeares agoe," that is, about 1621, and "I with mine owne hand made me two such Circles, which I have used ever since, as my occasions required," (*Epistle* p. (22)). On the same page, he describes his mode of discovery thus:

> I found that it required many times too great a paire of Compasses [in using Gunter's line], which would bee hard to open, apt to slip, and troublesome for use. I therefore first devised to have another Ruler with the former: and so by setting and applying one to the other, I did not onely take away the use of Compasses, but also make the worke much more easy and expedite: when I should not at all need the motion of my hand, but onely the glancing at my sight: and with one position of the Rulers, and view of mine eye, see not one onely, but the manifold proportions incident unto the question intended. But yet this facility also

wanted not some difficulty especially in the line of tangents, when one arch was in the former mediety of the quadrant, and the other in the latter: for in this case it was needful that either one Ruler must bee as long againe as the other; or else that I must use an inversion of the Ruler, and regression. By this consideration I first of all saw that if those lines upon both Rulers were inflected into two circles, that of the tangents being in both doubled, and that those two Circles should move one upon another; they with a small thread in the center to direct the sight, would bee sufficient with incredible and wonderfull facility to worke all questions of Trigonometry . . .

Oughtred said that he had no desire to publish his invention, but in the vacation of 1630 finally promised William Forster to let him bring out a translation. Oughtred claims that Delamain got the invention from him at Alhallontide [November 1], 1630, when they met in London. The accounts of that meeting we proceed to give in double column.

Delamain's Statement

Grammelogia IV, page (98)

". . . about Alhalontide 1630. (as our *Authors* reporteth) was the time he was *circumvented*, and then *his intent in a loving manner* (*as before*) *he opened unto me*, which particularly I will dismantle in the very naked truth: for, wee being walking together some few weekes before *Christmas*, upon *Fishstreet hill*, we discoursed upon sundry things *Mathematicall*, both *Theoreticall* and Practicall, and of the excellent inventions and helpes that in these dayes were produced, amongst which I was not a little taken with that of the *Logarythmes*, commending greatly the ingenuitie of Mr. Gunter in the *Projection*, and inventing of his *Ruler*, in the lines of proportion, extracted from these *Logarythmes* for ordinary *Practicall uses*; He replyed unto me (in these very words) What will yov say to an *Invention* that I have, which in a lesse extent of the *Compasses* shall worke truer then that of Mr. *Gunters Ruler*, I asked him then of what forme it was, he answered with some pause (which no doubt argued his suspition of mee that I might conceive it) that it was *Archingwise*, but now hee sayes that hee told mee then, it was *Circular* (but were I put to my oath to avoid the guilt of Conscience I would conclude in the former.) At which immediately I answered, I had the like my selfe, and so we discoursed not a word more touching that subject . . . Then after my coming home I sent him a sight of my *Projection* drawne in *Pastboard*: Now admit I had not the *Invention* of my *Ring* before I discoursed . . . it was not so facil for mee . . . to raise and compose so complete, and absolute an *Instrument* from so small a principle, or glimpse of light . . ."

Oughtred's Statement

Epistle, page (23)

"Shortly after my gift to *Elias Allen*, I chanced to meet with *Richard Delamain* in the street (it was at Alhallontide) and as we walked together I told him what an Instrument I had given to Master *Allen*, both of the Logarithmes projected into circles, which being lesse then one foot diameter would performe as much as one of Master *Gunters* Rulers of sixe feet long: and also of the Prostaphaereses of the Plannets and second motions. *Such an invention have I* said he: for now his *intentions* (that is his ambition) beganne to worke: . . . But he saith, *Then after my comming home I sent him a sight of my projection drawne in pastboard*. See how notoriously he jugleth without an Instrument. *Then after*: how long after? *a sight of my projection*: of how much? More then seven weekes after on December 23, he sent to mee the line of numbers onely set upon a circle: . . . and so much onely he presented to his Majesty: but as for Sine or tangent of his, there was not the least shew of any. Neither could he give to Master *Allen* any direction for the composure of the circles of his Ring, or for the division of them: as upon his oath Master *Allen* will testify how hee misled him, and made him labour in vain above three weeks together, until Master *Allen* himselfe found out his ignorance and mistaking, which is more cleare then is possible with any impudence to be outfaced."

Oughtred makes a further statement (*Epistle*, p. (24)) as follows:

> Delamain hearing that Brown with his *Serpentine* had *another line* by which he could worke to minutes in the 90 degree of sines . . . gave the [his] booke to Browne: who in thankfulnesse could not but gratify Delamain with his *Lines* also: and teach him the use of them, but especially of the *great Line*: with this caution on both sides, that one should not meddle with the others invention. Two dayes after *Delamain* . . . because he had found some things to be altered therin, . . . asked for the booke . . . but as soone as he had got it in his hands he rent out all the middle part with the two Schemes & put them up in his pocket & went his way . . . and . . . laboureth to recall all the bookes he had given forth . . . And shortly after this he got a new Printer (who was ignorant of his former Schemes) to print him new: giving him an especiall charge of the *outermost line newly graven* in the Plate, which indeed is *Brownes very line*: and then altering his book . . .

This and other statements made by Oughtred seem damaging to Delamain's reputation. But it is quite possible that Oughtred's guesses as to Delamain's motives are wrong. Moreover, some of Oughtred's statements are not first hand knowledge with him, but mere hearsay. One may accept his first hand facts and still clear Delamain of wrong doing. There is always danger that rival claimants of an invention or discovery will proceed on the assumption that no one else could possibly have come independently upon the same devices that they themselves did; the history of science proves the opposite. Seldom is an invention of any note made by only one man. We do not feel competent to judge Delamain's case. We know too little about him as a man. We incline to the opinion that the hypothesis of independent invention is the most plausible. At any rate, Delamain figures in the history of the slide rule as the publisher of the earliest book thereon and as an enthusiastic and skillful designer of slide rules.

The effect of this controversy upon interested friends was probably small. Doubtless few people read both sides. Oughtred says:[20] "this scandall . . . hath with them, to whom I am not knowne, wrought me much prejudice and disadvantage . ." Aubrey,[21] a friend of Oughtred, refers to Delamain "who was so sawcy to write against him" and remembers having seen "many yeares since, twenty or more good verses made" against Delamain. Another friend of Oughtred, William Robinson, who had seen some of Delamain's publications, but not his *Grammelogia IV*, wrote in a letter to Oughtred, shortly before the appearance of the latter's *Epistle*:

> I cannot but wonder at the indiscretion of Rich. Delamain, who being conscious to himself that he is but the pickpurse of another man's wit, would thus inconsiderately provoke and awake a sleeping lion . . . he hath so weakly (though in my judgment, vaingloriously enough) commended his own labour . . .[22]

Delamain presented King Charles I with one of his sun-dials, also with a manuscript and, later, with a printed copy of his book of 1630. A drawing of his improved slide rule was sent to the King and the *Grammelogia IV* is dedicated to him.

[20] *Epistle*, p. (8).

[21] Aubrey, *op cit.*, Vol. II., p. 111.

[22] Rigaud, *Correspondence of Scientific Men during the 17th Century*, Vol. I, Oxford, 1841, p. 11.

The King must have been favorably impressed, for Delamain was appointed tutor to the King in mathematics. His widow petitioned the House of Lords in 1645 for relief; he had ten children.[23]

Anthony Wood states that Charles I, on the day of his execution, commanded his friend Thomas Herbert "to give his son the duke of York his large ring-sundial of silver, a jewel his maj. much valued." Anthony Wood adds, "it was invented and made by Rich. Delamaine a very able mathematician, who projected it, and in a little printed book did shew its excellent use in resolving many questions in arithmetic and other rare operations to be wrought by it in the mathematics."[24]

VI. OUGHTRED'S GAUGING LINE, 1633

It has not been generally known, hitherto, that Oughtred designed a rectilinear slide rule for gauging and published a description thereof in 1633.[25] In his *Circles of Proportion*, chapter IX, Oughtred had offered a closer approximation than that of Gunter for the capacity of casks. The Gauger of London expostulated with Oughtred for presuming to question anything that Gunter had written. The ensuing discussion led to an invitation extended by the Company of Vintners to the instrument maker Elias Allen to request Oughtred to design a gauging rod.[26] This he did, and Allen received an order for "threescore" instruments. On page 19 Oughtred describes his 'Gauging Rod:'

> It consisteth of *two rulers of brasse* about 32 ynches of length, which also are halfe an ynch broad, and a quarter of an ynch thick . . . At one end of both those rulers are *two little sockets* of brasse fastened on strongly: by which the rulers are held together, and made to move one upon another, and to bee drawne out unto any length, as occasion shall require: and when you have them at the just length, there is upon one of the sockets *a long Scrue-pin* to scrue them fast.

There are graduations on three sides of the rulers, one graduation being the logarithmic line of numbers. He says (p. 39), "the maner of computing the *Gauge-divisions* I have concealed." W. Robinson, who was a friend of Oughtred, wrote him as follows:[27]

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[23] *Dictionary of National Biography*, Art. "Delamain, Richard." See also Rev. Charles J. Robinson, *Taylors' School, from A.D. 1562 to 1874*, Vol. I, 1882, p. 151; *Journal of the House of Commons*, Vol. IV., p. 197*b*; *Sixth Report of the Royal Commission on Historical Manuscripts*, Part I, Report and Appendix, London, 1877. In this *Appendix*, p. 82, we read the following:

> Oct. 22 [1645] Petition of Sarah Delamain, relict of Richard Delamain. Petitioner's husband was servant to the King, and one of His Majesty's engineers for the fortification of the kingdom, and his tutor in mathematical arts; but upon the breaking out of the war he deserted the Court, and was called by the State to several employments, in fortifying the towns of Northampton, Newport, and Abingdon; and was also abroad with the armies as Quartermaster-General of the Foot, and therein died. Petitioner is left a disconsolate widow with ten children, the four least of whom are now afflicted with sickness, and petitioner has nothing left to support them. There are several considerable sums of money due to the petitioner, as well from the King as the State. Prays that she may have some relief amongst other widows. See L. J., VII. 6. 657.

[24] Anthony Wood, *Athenae Oxonienses* (Edition Bliss) Vol. IV., London, 1820, p. 34.

[25] *The New Artificial Gauging Line or Rod: together with rules concerning the use thereof: Invented and written by WILLIAM OUGHTRED*, etc., London, 1633. The copy we have seen is in the Bodleian Library, Oxford. The book is small sized and has 40 pages.

[26] Oughtred, *op. cit.*, p. 11.

[27] S. J. Rigaud, *Correspondence of Scientific Men of the 17th Century*, Oxford, Vol. I, 1841, p. 17.

I have light upon your little book of artificial gauging, wherewith I am much taken, but I want the rod, neither could I get a sight of one of them at the time, because Mr. Allen had none left . . . I forgot to ask Mr. Allen the price of one of them, which if not much I would have one of them." Oughtred annotated this passage thus: "Or in wood, if any be made in wood by Thompson or any other."

Another of Oughtred's admirers, Sir Charles Cavendish, wrote, on February 11, 1635 thus:[28]

I thank you for your little book, but especially for the way of calculating the divisions of your gauging rod. I wish, both for their own sakes and yours, that the citizens were as capable of the acuteness of this invention, as they are commonly greedy of gain, and then I doubt not but they would give you a better recompense than I doubt now they will.

On April 20, 1638, we find Oughtred giving Elias Allen directions[29] "about the making of the two rulers." As in 1633,[30] so now, Oughtred takes one ruler longer than the other. This 1633 instrument was used also as "a crosse-staffe to take the height of the Sunne, or any Starre above the Horizon, and also their distances." The longer ruler was called *staffe,* the shorter *transversarie.* While in 1633 he took the lengths of the two in the ratio "almost 3 to 2," in 1638, he took "the transversary three quarters of the staff's length, . . . that the divisions may be larger."

VII. OTHER SEVENTEENTH CENTURY SLIDE RULES

In my *History of the Slide Rule* I treat of Seth Partridge, Thomas Everard, Henry Coggeshall, W. Hunt and Sir Isaac Newton.[31] Of Partridge's *Double Scale of Proportion,* London, I have examined a copy dated 1661, which is the earliest date for this book that I have seen. As far as we know, 1661 is the earliest date of publications on the slide rule, since Oughtred and Delamain. But it would not be surprising if the intervening 28 years were found not so barren as they seem at present. The 1661 and 1662 impressions of Partridge are identical, except for the date on the title-page. William Leybourn, who printed Partridge's book, speaks in high appreciation of it in his own book.[32]

In 1661 was published also John Brown's first book, *Description and Use of a Joynt-Rule,* previously mentioned. In Chapter XVIII he describes the use of "Mr. Whites rule" for the measuring of board and timber, round and square. He calls this a "sliding rule." The existence, in 1661, of a "Whites rule" indicates activities in designing of which we know as yet very little. In his book of 1761, previously quoted, Brown gives a drawing of "White's sliding rule" (p. 193); also a special contrivance of his own, as indicated by him in these words:

A further improvement of the Triangular Quadrant, as I have made it several times, with a sliding Cover on the in-side, when made hollow, to carry Ink, Pens, and Compasses; then on the sliding Cover, and Edges, is put the Line of Numbers, according to Mr. White's first Contrivance for manner of operation; but much augmented, and made easie, by John Brown.

[28] Rigaud, *loc. cit.*, p. 22.
[29] Rigaud, *loc. cit.*, pp. 30, 31.
[30] Oughtred, *An Addition vnto the Vse of the Instrument called the Circles of Proportion*, London, 1633, p. 63.
[31] F. Cajori, *History of the Slide Rule*, New York, 1909, pp. 16–22, Addenda, pp. vi–ix
[32] W. Leybourn, *op. cit.*, 1673, Preface, and pp. 128–29.

He gives no drawing of his "triangular quadrant," hence his account of it is unsatisfactory. He explains the use of "gage-points." His placing logarithmic lines on the edges of instrument boxes was outdone in oddity later by Everard who placed them on tobacco-boxes.[33] In Brown's publication of 1704 the White slide rule is given again, "being as neat and ready a way as ever was used." He tells also of a "glasier's sliding rule." William Leybourn explains in 1673 how Wingate's double and triple lines for squaring and cubing, or square and cube root, can be used on slide rules.[34]

Beginning early in the history of the slide rule, when Oughtred designed his "gauging rod," we notice the designing of rules intended for very special purposes. Another such contrivance, which enjoyed long popularity, was the *Timber Measure by a Line*, by Hen. Coggeshall, Gent., London, 1677, a booklet of 35 pages. Coggeshall says in his preface:

> For what can be more ready and easie, then having set twelve to the length, to see the Content exactly against the Girt or Side of the Square. Whereas on Mr. Partridge's Scale the Content is the Sixth Number, which is far more troublesome then [even] with Compasses.

One line on Coggeshall's rule begins with 4 and extends to 40, these numbers being the "Girt" (a quarter of the circumference), which in ordinary practice of measuring round timber lies between 4 inches and 40 inches. This "Girt line" slides "against the line of Numbers in two Lengths, to which it is exactly equal." A second edition, 1682, shows some changes in the rule, as well as an enlargement and change of title of the book itself: *A Treatise of Measures, by a Two-foot Rule*, by H. C. Gent, London, 1682. In this, the description of the rule is given thus:

> There are four Lines on each flat of this Rule; two next the outward edges, which are Lines of Measure; and two next the inward edges, which are Lines of Proportion. On one flat, next the inward edges, is the Square-line [Girt-line in round timber measurement] with the Line of Numbers his fellow. Next the outward, a Line of Inches divided into Halfs, Quarters, and Half-Quarters; from 1 to 12 on one Rule; and from 12 to 24 on the other. On the other flat, next the inward edges, is the double Scale of Numbers [for solving proportions]. Next the outward on one Rule a Line of Inches divided each into ten parts; and this for gauging, etc. On the other a foot divided into 100 parts.

Later further changes were introduced in Coggeshall's rule.[34]

It is worthy of note that Coggeshall's slide rule book, *The Art of Practical Measuring*, was reviewed in the *Acta eruditorum*, anno 1691, p. 473; hence Leupold's description[35] of the rectilinear slide rule in his *Theatrum arithmetico-geometricum*, Leipzig, 1727, Cap. XIII, p. 71, is not the earliest reference to the rectilinear rule found in German publications. The above date is earlier even than Biler's reference to a circular slide rule in his *Descriptio instrumenti mathematici universalis* of 1696.

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[33] Cajori *op. cit.*, Addenda, p. ix.

[34] William Leybourn, *op. cit.*, 1673, p. 35.

[34] See Cajori, *op. cit.*, pp. 20, 28, Addenda, p. ix.

[35] See F. Cajori, "A Note on the History of the Slide Rule," *Bibliotheca mathematica*, 3 F., Vol. 10, pp. 161–163.

Two noted slide rules for gauging were described by Tho. Everard, Philomath, in his *Stereometry made easie*, London, 1684. He designates his lines by the capital letters A, B, C, D, E. On the first instrument, *A* on the rule, and *B* and *C* on the slide, have each two radiuses of numbers, *D* has only one, while *E* has three. The second rule is described in an *Appendix;* it is one foot long, with two slides enabling the rule to be extended to 3 feet.

Everard's instruments were made in London by Isaac Carver who, soon after, himself wrote a sixteen-page *Description and Use of a New Sliding Rule, projected from the Tables in the Gauger's Magazine*, London, 1687, which was "printed for William Hunt" and bound in one volume with a book by Hunt, called *The Gauger's Magazine*, London, 1687. This appears to be the same William Hunt who later brought out descriptions of his own of slide rules. The instrument described by Carver "consists of three pieces, two whereof are moveable to be drawn out till the whole be 36 inches long." It has several non-logarithmic graduations, together with logarithmic lines marked A, B, C, D, of which A, B, C are "double lines," and D a "single line" used for squares and square roots. It is designed for the determination of the vacuity of a "spheroidal cask lying," a "spheroidal cask standing," and a "parabolical cask lying."

Another seventeenth century writer on the slide rule is John Atkinson, whom we have mentioned earlier. He says:[36] "The Lines of Numbers, Sines and Tangents, are set double, that is, one on each side, as the middle piece slides: which middle piece is so contrived, to slip to and fro easily, to slide out, and to be put in any side uppermost, in order to bring those Lines together (or against one another) most proper for solving the Question, wrought by *Sliding-Gunter*."

The data presented in this article show that, while the earliest slide rules were of the circular type, the later slide rules of the seventeenth century were of the rectilinear type.[37]

January 12, 1915.

[36] John Atkinson, *op. cit.*, 1694, p. 204.

[37] Probably the oldest slide rule now in existence is owned by St. John's College, Oxford, and is in the form of a brass disc, 1 ft. 6 in. in diameter. It was exhibited along with other instruments in May, 1919. According to the *Catalogue of a Loan Exhibition of Early Scientific Instruments* in Oxford, opened May 16, 1919, the instrument is inscribed with the name of the maker ("*Elias Allen fecit*") and with the name of the donor, Georgius Barkham. It is dated 1635, which is only three years after the first publication of Oughtred's description of his circular slide rule. It is stated in the *Catalogue:* "Unfortunately all the movable parts but the base-plate and a couple of thumb-screws are missing. The face of the instrument is engraved with Oughtred's *Horizontal Instrument.* The back is engraved with eleven Circles of Proportion as described in Arthur Haughton's book, a copy of which was presented to St. John's College by George Barkham, to explain the use of the instrument." As Arthur Haughton's Oxford edition of Oughtred's *Circles of Proportion* did not appear until 1660, it would seem that the instrument was probably not presented to the College before 1660. As far as is known, the next oldest slide rule is of the year 1654, kept in the South Kensington Museum, London, and is described in *Nature* of March 5, 1914. It is a rectilinear rule, "of boxwood, well made, and bound together with brass at the two ends. It is of the square type, a little more than 2 ft. in length, and bears the logarithmic lines first described by Edmund Gunter. Of these, the *num, sin* and *tan* lines are arranged in pairs, identical and contiguous, one line in each pair being on the fixed part, and the other on the slide." The instrument is inscribed, "Made by Robert Bissaker for T. W., 1654." Nowhere else have we seen reference to Robert Bissaker. His slide rule seems to antedate the "Whites rule." mentioned above. [This foot-note was added on October 15, 1919.]